South China Morning Post

Source:
https://www.scmp.com/news/china/science/article/3286686/ancient-chinese-proved-pythagorean-theorem-17-characters-mathematician

China / Science

Ancient Chinese proved the Pythagorean theorem in 17 characters: mathematician

A leading academic has highlighted China's rich mathematical history, including a startling fact about a well-known theorem

Zhang Tong in Beijing
Published: 5:22pm, 22 Nov 2024 / Updated: 11:24am, 28 Nov 2024

Pythagorean theorem was proved hundreds of years before Euclid by an ancient Chinese mathematician. Photo: Shutterstock

Hundreds of years before a Greek scholar outlined his proof of the Pythagorean theorem, ancient Chinese scholars proved it using just 17 characters, a renowned Chinese mathematician highlighted in a recent lecture.

Zhou Xiangyu, a member of the Chinese Academy of Sciences (CAS) and former director of its Institute of Mathematics, delivered the lecture at the National Science Communication Centre last Friday, where he spoke about China's rich mathematical heritage.

"Ancient Chinese mathematics is a treasure trove, concise in language yet profound in meaning. Many more insights await discovery," Zhou told Science and Technology Daily on Sunday.

He believes ancient Chinese books contain theories that significantly influenced modern mathematics and contributed to Chinese language and culture. Among these theories, the proof of the Pythagorean theorem is particularly fascinating.

The theorem defines the relationship between the three sides of a right-angled triangle, stating that the square of the hypotenuse equals the sum of the squares of the other two sides.

In Western history, the earliest written proof appears in *Elements* by Euclid, a Greek mathematician in the 4th century BC.

But through his research, Zhou found that a complete definition and proof of the theorem existed as early as the Western Zhou dynasty in China (around 1000BC).

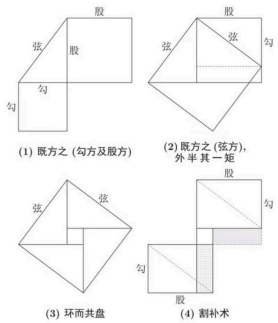
This proof appears in the opening of *The Mathematical Classic of Zhou*, known as the *Zhoubi Suanjing*, in a dialogue between Zhou Gong, who was a foundational figure in Confucianism, and Shang Gao, a

mathematician from the Western Zhou period.

In it, Zhou Gong asks Shang Gao: "The sky cannot be climbed step by step, nor can the Earth be measured by a ruler – so how can we measure the size of heaven and Earth?"

In response, Shang Gao suggests using mathematical methods to abstract and summarise things, taking the Pythagorean theorem as an example, offering both the definition and proof.

The original text contains just 17 characters: "*Ji fang zhi, wai ban qi yi ju, huan er gong pan, de cheng san si wu*," which roughly means: "Draw a square around them, halve one rectangle side, rotate it all around, and you get three, four and five."



An illustrated depiction of Shang Gao's proof of Pythagorean theorem. This figure is the basis for the emblem of the Institute of Mathematics and Systems Science at CAS. Credit: Zhou Xiangyu

Zhou Xiangyu noted in the interview that he spent several years studying historical documents and reflecting on the meaning of each character. He eventually realised that "*ji*" meant "all" or "entire" – shedding light on its use in this context.

In 2022, Zhou published a paper in Acta Mathematica Sinica outlining two interpretations of this phrase.

The first interpretation uses three squares for each triangle side, dividing the second largest square in half, then rotating the resulting shapes to create a windmill-like pattern. Using a technique of rearranging areas, the theorem can be proved visually.

The second approach follows the same steps initially, but changes the orientation of the triangle during rotation, which makes the final calculation of area more intuitive.

Both interpretations demonstrate that ancient Chinese mathematical thought already integrated algebra with geometry. Remarkably concise, the classical Chinese proof uses just 17 characters, compared to about 10 lines of geometric formulas in Euclid's explanation.

To Zhou, these characters provide a rigorous scientific explanation, challenging the misconception that Chinese mathematics lacked theoretical proofs.

"Shang Gao's proof of the Pythagorean theorem is a scientific fact that deserves recognition," Zhou told Science and Technology Daily. "Even when examined through the lens of modern mathematics, his proof holds up."

An earlier article by Qu Anjing, history of science professor at Northwestern University in Xian, also argued that Shang Gao indeed proved the Pythagorean theorem.

He pointed out that in about the 3rd century, mathematician Zhao Shuang wrote an extensive commentary on *The Mathematical Classic of Zhou* and added an illustration confirming the theorem, which has since become widely known.

This illustration, widely admired for its elegance, inspired the emblem of the Institute of Mathematics and Systems Science at CAS and was used as the emblem for the 24th International Congress of Mathematicians held in Beijing in 2002.

Zhou was mentored by a student of the legendary mathematician Hua Luogeng. Outside his professional work, Zhou studies the history of mathematics and has given many public lectures.

Hua Luogeng said, "Mathematics should be studied both broadly and deeply," a view Zhou Xiangyu strongly agrees with.

"In studying mathematics, we should pay attention to the origins and development of mathematical concepts and methods, the roots and foundations, the source and flow."
